

Overview of Trigonometric Functions Values and Basic Facts

If r is the radius of a circle and θ is an angle measured from standard position, then we can find the corresponding location on the edge of the circle by using the formulas

$$x = r \cos(\theta) = r \cos(\theta_0 \pm wt) \quad \text{and} \quad y = r \sin(\theta) = r \sin(\theta_0 \pm wt)$$

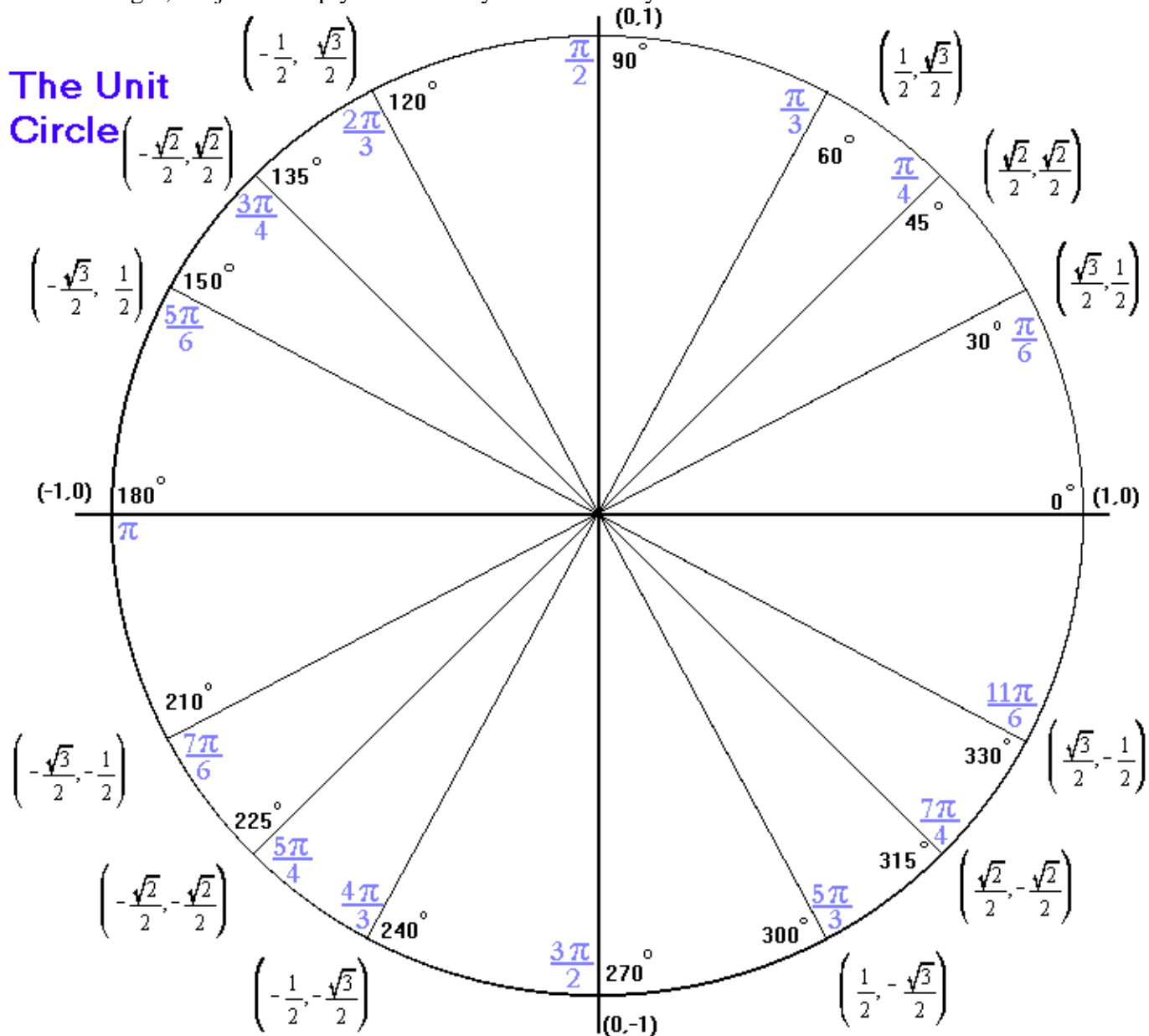
For most values of θ , $\sin(\theta)$ and $\cos(\theta)$ are not easily computed and require a calculator. However, you are expected to know the following values:

Angle		$\sin(\theta)$	$\cos(\theta)$
0 deg	0 rad	0	1
30 deg	$\pi/6$ rad	$1/2$	$\sqrt{3}/2$
45 deg	$\pi/4$ rad	$\sqrt{2}/2$	$\sqrt{2}/2$
60 deg	$\pi/3$ rad	$\sqrt{3}/2$	$1/2$
90 deg	$\pi/2$ rad	1	0

You can find the other trig function values at these angles using the relationships:

$$\tan(\theta) = \sin(\theta)/\cos(\theta), \quad \cot(\theta) = \cos(\theta)/\sin(\theta), \quad \csc(\theta) = 1/\sin(\theta), \quad \sec(\theta) = 1/\cos(\theta).$$

Often these values are remembered by actually putting them on a circle. Here is the circle with radius 1 (or the *unit circle*) with the values at the above angles label along with corresponding angles in other quadrants. If the radius is larger, we just multiply each x and y coordinates by the radius.



Here are some useful relationships between the trig functions.

1. $\sin^2(\theta) + \cos^2(\theta) = 1$ and dividing by $\cos^2(\theta)$ gives $\tan^2(\theta) + 1 = \sec^2(\theta)$
so if we know $\cos(\theta)$ we can solve for $\sin(\theta)$ or vice versa.
2. $\cos(-\theta) = \cos(\theta)$
 $\sin(-\theta) = -\sin(\theta)$
3. $\cos(\theta) = \cos(\theta + 2\pi n)$ for any whole positive or negative whole number (integer) n .
 $\sin(\theta) = \sin(\theta + 2\pi n)$ for any whole positive or negative whole number (integer) n .
In other words, the function values repeat every 2π (we say the functions are *periodic*).
4. $\sin(\theta) = \cos(\theta - \pi/2)$ (shift Cosine right by $\pi/2$ and you get Sine)
 $\cos(\theta) = \sin(\theta + \pi/2)$ (shift Sine left by $\pi/2$ and you get Cosine)
5. Addition and Subtraction Identities (we don't use these much in this class, but you will use these occasionally in Math 124 and more often in Math 125).
 $\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
 $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
In particular, if $a = b = x$, then you get the often used double angle identities:
 $\sin(2x) = 2\sin(x)\cos(x)$
 $\cos(2x) = \cos^2(x) - \sin^2(x)$
And combining this last fact with the fact 1 above, you can solve to find the half angle identities:
 $\sin^2(x) = (1 - \cos(2x))/2$
 $\cos^2(x) = (1 + \cos(2x))/2$

For this class, we primarily make use of the first 4 identities above. All the identities you need for calculus will be in the front cover of your calculus book, but the list above will typically get you by.

One other useful thing to remember, when you are working with a trig function you can always draw a triangle visualizing the relationship you are dealing with. For example, if you know that $\tan(\theta) = 2/3$, that means θ corresponds to a right triangle that has opposite side length equal to 2 and adjacent side length equal to 3. Hence we would be able to use the Pythagorean Theorem to find the hypotenuse and from that we could give any of the other trig values. These are techniques that will arise in calculus from time to time.

For now, our focus is on circular motion and being able to graph the Sine and Cosine waves. Hopefully, you are getting a good grasp of circular motion namely:

1. Find the radius
2. Find the initial angle
3. Find angular speed
4. Make angular speed positive or negative depending on if the motion is counterclockwise or clockwise (respectively).
5. Plug in these facts to: $x = r \cos(\theta_0 \pm \omega t)$ and $y = r \sin(\theta_0 \pm \omega t)$

You will not always do these steps in this order, but the goal is to find the three pieces of information to fill in this model. Then you can plug in any time and find your location on the circle.